TEMPERATURE TRANSIENT AFTER CONTACT HEATING

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THIS note gives equations representing transient temperature distributions in two heat-conducting, one-dimensional finite solid bars following contact heating. Both bars are assumed to have identical uniform physical properties which remain constant during the transient. The bars are subject to the following conditions (Fig. 1).

(1) Bar 1 with a length H is initially at a constant temperature T_1'.
(2) Bar 2 with a length $L - H$ is initially at a constant temperature $T_2'(T_1' \neq T_2')$.
(3) Bars are brought into contact at time, $t = 0$. Heat is conducted only through the contact surface.
(4) Contact is discontinued at time, $t = t_0$. No heat is transferred to the surroundings.

One dimensional heat conduction is represented by,

$$\frac{\partial T'}{\partial t} = k \frac{\partial^2 T'}{\partial Z^2} \qquad (1)$$

where T' is the temperature, Z is the distance variable and k the thermal diffusivity. The solution of equation (1)

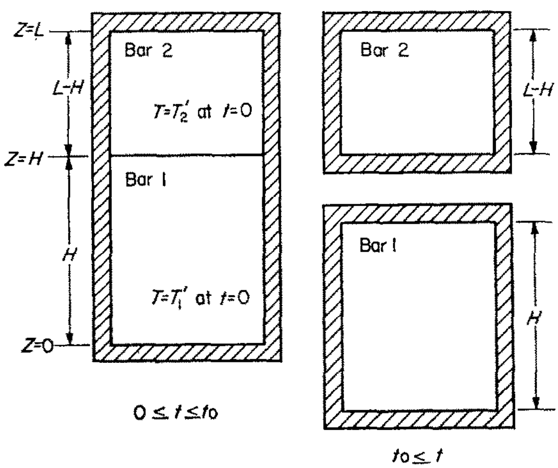

FIG. 1. Contact heat conduction between two solid bars.

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for $0 \leqslant t < t_0$ and subject to the conditions stated above is given by [1],

$$T = X + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp(-n^2\pi^2\tau)\sin(n\pi X)\cos(n\pi\sigma) \qquad (2)$$

where

$$\sigma = \frac{Z}{L}, \quad \tau = \frac{kt}{L^2}, \quad T = \frac{T' - T_2'}{T_1' - T_2'}, \text{ and } X = \frac{H}{L}.$$

The solutions of the conduction equation representing the temperature transient after discontinuing contact ($t \geqslant t_0$ or $\tau \geqslant \tau_0$) are derived as follows:

Bar 1:

$$\left.\begin{aligned} \frac{T}{X} = 1 - \frac{2}{\pi^2 X^2}\sum_{n=1}^{\infty}\frac{\exp(-n^2\pi^2\tau_0)}{n^2} \\ \sin^2 n\pi X - \frac{2}{\pi^2 X}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{n} \\ \exp(-n^2\pi^2\tau_0)(\sin n\pi X)[\exp(-m^2\pi^2\tau_1)] \\ (\cos m\pi\sigma_1)\left[\frac{\sin\pi(nX - m)}{nX - m} + \frac{\sin(nX - m)\pi}{nX - m}\right]. \end{aligned}\right\} \qquad (3)$$

Bar 2:

$$\left.\begin{aligned} \frac{T}{X} = 1 - \frac{2}{\pi^2 X(1 - X)}\sum_{n=1}^{\infty}\frac{\exp(-n^2\pi^2\tau_0)}{n^2} \\ \sin^2 n\pi X - \frac{4(1 - X)}{\pi^2 X}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{n} \\ \exp(-n^2\pi^2\tau_0)(\sin^2 n\pi X)[\exp(-m^2\pi^2\tau_2)] \\ (\cos m\pi\sigma_2)\left[\frac{1}{n^2(1 - X)^2 - m^2}\right] \end{aligned}\right\} \qquad (4)$$

where

$$\sigma_1 = \frac{\sigma}{X} = \frac{Z}{H}, \quad \tau_1 = \frac{kt}{H^2}$$

$$\sigma_2 = \left(\frac{\sigma - X}{1 - X}\right) = \frac{Z}{L - H}, \quad \tau_2 = \frac{kt}{(L - H)^2}.$$

Fig. 2 illustrates some results of numerical computation.

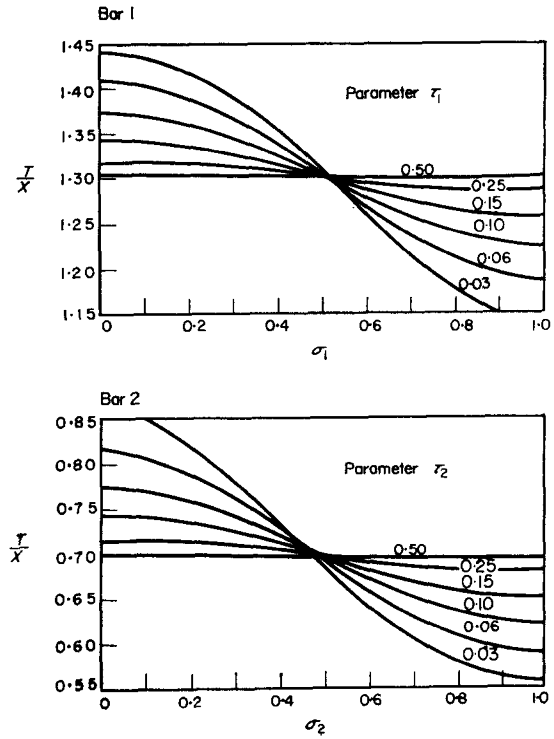

FIG. 2. Temperature distribution represented by equations (3) and (4); $\tau_0 = 0{\cdot}10$.

The equations derived are equally valid for molecular diffusion.

REFERENCES

1. H. S. CARSLAW and J. C. JAEGER, *Conduction of Heat in Solids*, 2nd Ed. Oxford University Press (1959).

THE TRANSITION FROM BLACK BODY TO ROSSELAND FORMULATIONS IN OPTICALLY THICK FLOWS†

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THE energy radiated by optically thick flows through their boundaries is expressed sometimes as a black body flux $q^R = \sigma T^4$, sometimes as a Rosseland diffuse flux

$$q^R = -\frac{16\sigma T^3}{3k_R}\frac{\partial T}{\partial y}.$$

The purpose of this note is to establish, on the basis of two simple incompressible flows, the radiation parameters which govern the choice between these two formulations.

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